Page no-319

Example 5-11 Calculating nonuniform basis functions

Calculate the five (n+1 = 5) third – order (k = 3) basis functions Ni,3 (t) , i=1, 2,3, 4, 5 using the knot vector [X] = [0 0 0 1 1 3 3 3] which contains an interior repeated knot value.

SOLUTION-

$0 \le t < 1$

$N_{3,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 3$

$N_{2,2}(t) = 1-t$; $N_{3,2}(t) = t$; $N_{i,2}(t) = 0$, $i \neq 2,3$

$N_{1,3}(t) = (1-t)^2$; $N_{2,3}(t) = t(1-t) + (1-t)t = 2t(1-t)$;

$N_{3,3}(t) = t^2$; $N_{i,3}(t) = 0$, $i \neq 1,2,3$

$1 \le t < 1$

$N_{i,1}(t) = 0$, all i

$N_{i,2}(t) = 0$, all i

$N_{i,3}(t) = 0$, all i

Notice specifically that as a consequence of the multiple knot value, $N_{4,1}(t) = 0$ for all t.

$1 \le t < 3$

$N_{5,1}(t) = 1; \quad N_{i,1}(t) = 0, \ i \neq 5$

$N_{4,2}(t) = (3-t)/2 \ ; \quad N_{5,2}(t) = (t-1)/2 \ ; \ N_{i,2}(t) = 0, \ i \neq 4,5$

$N_{3,3}(t) = (3-t)^2/4 \ ;$

$N_{4,3}(t) = (t-1)(3-t)/4 + (3-1)(t-1)/4 = (3-t)(t-1)/2 \ ;$

$N_{5,3}(t) = (t-1)^2/4 \ ; \ N_{i,3}(t) = 0, \ i \neq 3, 4,5$

The results are shown in figure 1.

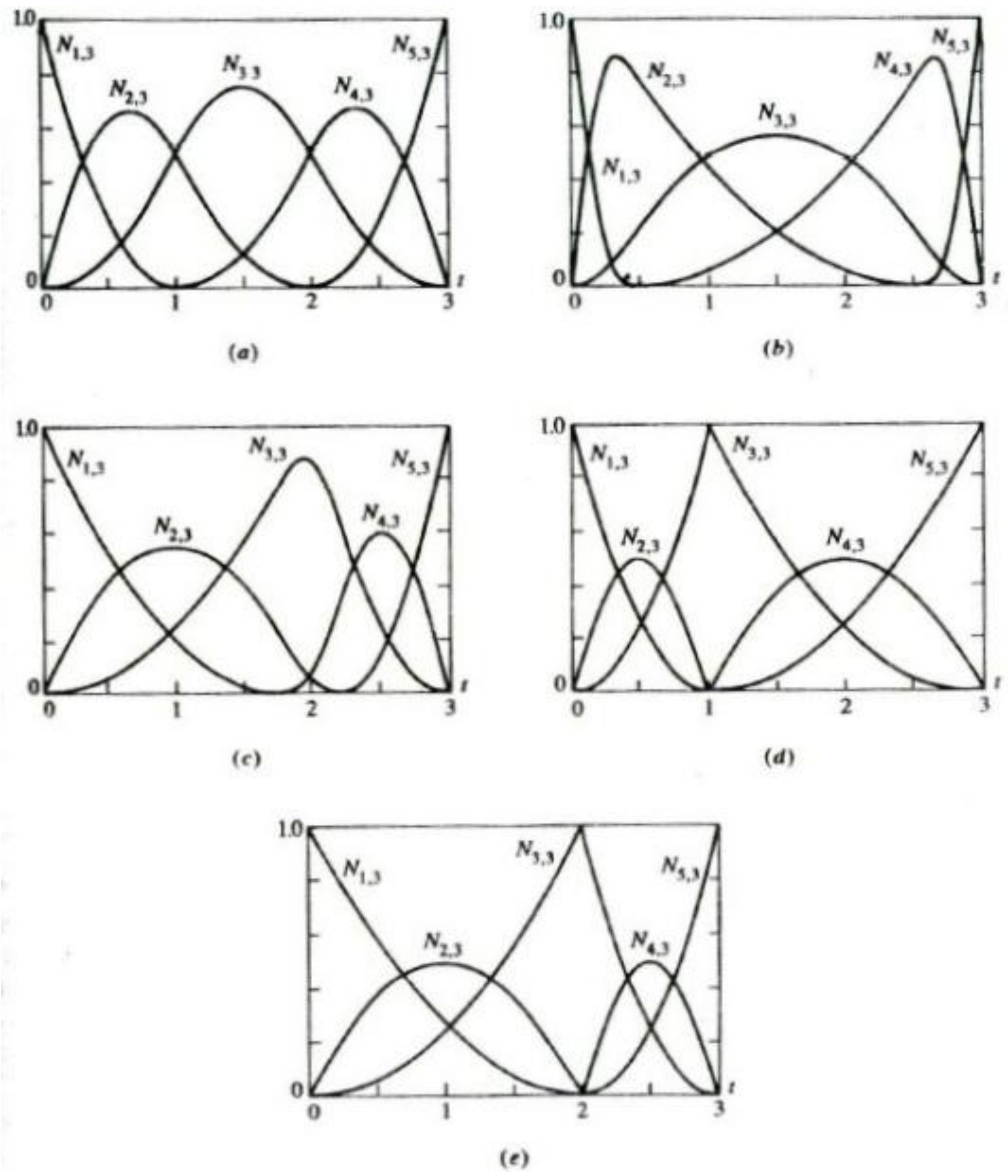

Figure.1

Notice that for each value of t the $\sum N_{i,k}(t) = 1.0$. For example, with

$0 \le t < 1$

$$\sum_{i=1}^{5} N_{i,3}(t) = (1-t)^2 + 2t(1-t) + t^2$$

$$= 1-2t+t^2+2t-2t^2+t^2 = 1$$

Similarly, for $1 \le t < 3$

$$\sum_{i=1}^{5} N_{i,3}(t) = \frac{1}{4}[(3-t)^2 + 2(3-t)(t-1) + (t-1)^2]$$

$$= \tfrac{1}{4}[9-6t+t^2-6t+8t-2t^2+1-2t+t^2]$$

$$= 4/4 = 1$$

Page no-322

Example 5-12 calculating an open B-Spline Curve

Given B_1 [1 1] , B_2 [2,3], B_3[4,3] and B_4[3,1] the vertices of a Bezier polygon. Calculate both second and fourth – order B – Spline curves.

For k= 2 the open knot vector is

[0 0 1 2 3 3]

Where $x_1 = 0$, $x_2 = 0$, ……, $x_6 = 3$. The parameter range is $0 \le t \le 3$.the curve is composed of three linear ($k-1 = 1$) segments. For $0 \le t \le 3$ the basis functions are :

$0 \le t < 1$

$N_{2,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \ne 2$

$N_{1,2}(t) = 1-t$; $N_{2,2}(t) = t$; $N_{i,2}(t) = 0$, $i \ne 1,2$

$1 \le t < 2$

$N_{3,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \ne 2$

$N_{2,2}(t) = 2-t$; $N_{3,2}(t) = (t-1)$; $N_{i,2}(t) = 0$, $i \ne 2,3$

$2 \le t < 3$

$N_{4,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \ne 4$

$N_{3,2}(t) = (3-t)$; $N_{4,2}(t) = (t-2)$; $N_{i,2}(t) = 0$, $i \ne 3,4$

Using equation { $P_1'(0) = -P_n'(t_n)$ } the parametric B-Spline curve is

$$P(t) = B_1 N_{1,2}(t) + B_2 N_{2,2}(t) + B_3 N_{3,2}(t) + B_4 N_{4,2}(t)$$

For each of these intervals the curve is given by

$$P(t) = (1-t)B_1 + tB_2 = B_1 + (B_2 - B_1)t \quad 0 \le t < 1$$

$P(t) = (2-t)B_2 + (t-1)B_3 = B_2 + (B_3 - B_2)t$
$1 \le t < 2$

$P(t) = (3-t)B_3 + (t-2)B_4 = B_3 + (B_4 - B_3)t$
$2 \le t < 3$

In each case the result is the equation of the parametric straight line for the polygon span, i.e., the 'curve' is the defining polygon.

The last point on the curve ($t = t_{max} = 3$) requires special consideration .because of the open right – hand interval in equation

$$N_{(i,1)}(t), i = \begin{cases} 1, i, \text{if } x_i \le t < x_{i+1} \\ 0, otherwise \end{cases}$$

All the basis function $N_{i,K}$ at $t = 3$ are zero . consequently , the last polygon point does not technically lie on the B-Spline curve . however , pratically it does . consider $t = 3-\varepsilon$ .where ε is an infinitesimal value . letting $\varepsilon \rightarrow 0$ shows that in the limit the last point on the curve and the last polgon point are coincident . practically , this result is incorporated by either artificially adding the last polygon point to the curve description or by defining $N(t = t_{max}) = 1.0$.

For k=4 the order of the curve is equal to the number of defining polygon vertices. Thus the B-spline curve reduces to a bezier curve . the knot vector with t_{max} = n-k+2 = 3-4+2 =1 is [0 0 0 0 1 1 1 1].the basis functions are

$0 \leq t < 1$

$N_{4,1}(t) = 1$; $N_{i,1}(t) = 0$, $i \neq 4$

$N_{3,2}(t) = (1-t)$; $N_{4,2}(t) = t$; $N_{i,2}(t) = 0$, $i \neq 3,4$

$N_{2,3}(t) = (1-t)^2$; $N_{3,3}(t) = 2t(1-t)$;

$N_{4,3}(t) = t^2$; $N_{i,3}(t) = 0$, $I \neq 2,3,4$

$N_{1,4}(t) = (1-t)^3$; $N_{2,4}(t) = t(1-t)^2 + 2t(1-t)^2 = 3t(1-t)^2$;

$N_{3,4}(t) = 2t^2(1-t) + (1-t)t^2 = 3t^2(1-t)$; $N_{4,4}(t) = t^3$;

Using equation { $P_1'(0) = -P_n'(t_n)$ } the parametric B-Spline is

$P(t) = B_1 N_{1,4}(t)+B_2 N_{2,4}(t)+B_3 N_{3,3}(t)+B_4 N_{4,4}(t)$

$P(t) = (1-t)^3B_1 +3t(1-t)^2B_2 +3t^2(1-t)B_3 +t^3B_4$

Thus, at t=0

$P(0) = B_1$

And at t= ½

$P(1/2) =(1/8)B_1 +(3/8)B_2 + (3/8) B_3 + (1/8) B_4$

and

P(1/2) = 1/8 [1 1]+3/8 [2 3] + 3/8 [4 3] +1/8 [3 1]

= [11/4   5/2]

The resulting curve is shown in figure.2.

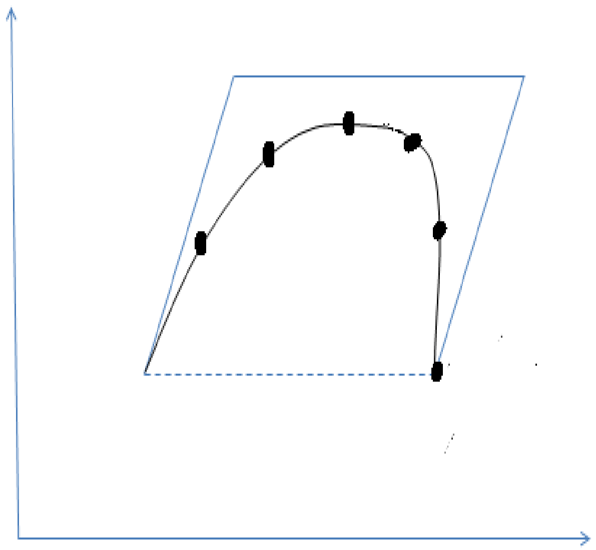

figure.2.